Multi-Feedback Successive Interference Cancellation with Dynamic Log-Likelihood-Ratio Based Reliability Ordering

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Abstract—In this paper, we propose a successive interference cancellation (SIC) detector which utilises multiple feedback (MF) candidates for cancellation stages and a dynamic log-likelihood-ratio (LLR)-based dynamic reliability ordering (RO) of the cancellation stages in a multiple-input multiple-output (MIMO) system. The proposed multi-feedback reliability-ordered SIC (MF-RO-SIC) detector’s performance is compared with VBLAST based techniques for MF, the dynamic LLR RO without MF and the maximum likelihood (ML) detector. The results show that the MF-RO-SIC can outperform the MF-SIC and the RO-SIC in bit error rate (BER) performance, with the ability to tune the performance by altering the shadowing criterion values.

Index Terms—interference cancellation, multiple feedback, reliability ordering, MIMO Systems

I. INTRODUCTION

Successive Interference Cancellation (SIC) detectors [1] are a well known class of detectors for multi-user or multiple-input multiple-output (MIMO) systems, where upon the estimation of a data symbol, the estimated data symbol’s effect is removed from the received signal in order to improve the signal to interference and noise ratio (SINR) of the remaining data symbols in that time instant. However, this type of detector can suffer from error propagation. If the estimated data symbols are not reliable then this unreliability can impair all subsequent symbol estimates at a given time instant, degrading the bit error rate (BER) performance.[2], [3], [4]

A well-known technique for improving the performance of the SIC detector is to remove the data streams with the greatest power first, thus removing the greatest source of interference first, which is known as the VBLAST technique [5], and is implemented by considering the powers of the channels associated with each user or antenna. However, as shown in [6], although the VBLAST cancellation order within the SIC is the optimal for the vast majority of detected symbols, other cancellation orders can outperform the VBLAST ordering on a given occasion, which could result in performance gains.

In this paper dynamic ordering (RO) based upon log-likelihood-ratios (LLR) [7] is considered in conjunction with a method of multiple-feedback (MF), [8] which is designed to provide alternative cancellation candidates for data symbols, to compensate for and correct potentially erroneous cancellation error propagation. These methods are integrated into the proposed MF-RO-SIC detector, which utilises both methods and makes considerations for reducing the computational complexity associated with these methods, whilst increasing the BER performance.

The organisation of this paper is as follows: Section II will detail the MIMO system model and MMSE detection, in Section III interference cancellation techniques for MIMO will be reviewed, Section IV will show the proposed MF-RO-SIC detector, and how the complexity of the proposed detector can be reduced. The simulation results and conclusions are given in Section V and Section VI, respectively.

II. SYSTEM MODEL AND MMSE DETECTION

The system under consideration is an uncoded point-to-point spatial multiplexing MIMO link, consisting of $N$ transmit antennas and $M$ receive antennas. At each time instant, the $N$ length column vector $x$ consisting of $N$ data symbols taken from the constellation set $V$ that is appropriate for the modulation scheme being used, is transmitted through the $M \times N$ channel matrix $H$. At the $M$ receive antennas at the destination, the transmitted signal vector is received as the $M$ length column vector $y$. This can be described as:

$$y = Hx + n$$  

where $n$ is the circular complex additive white Gaussian noise (AWGN) vector representing noise at the receive antennas with a variance:

$$\sigma^2 = M/\{2\gamma\}$$

where $\gamma$ is the signal-to-noise ratio (SNR). The channel $H$ is modelled as a complex Rayleigh distributed channel, and can be expressed as the horizontal concatenation of the individual channel vector associated with each transmit antenna,

$$H = [h_1, h_2, \ldots, h_N]$$

Minimum Mean Squared Error (MMSE) detection is well known, and uses a Wiener filter at the receiving MIMO device to retrieve an estimate of the originally transmitted symbols by the transmitting device, as shown below for a vector basis of the estimated symbols ($z$) and per-antenna basis ($z_n$):

$$z = W^H y, z_n = w_n^H y, n = 1, \ldots, N$$

where $W$ is the Wiener filter, calculated for a MMSE criterion as:

$$W = R_y^{-1}H$$
where $\mathbf{R}_n$ is the auto-correlation matrix of the received signal, and is expressed as:

$$
\mathbf{R}_y = \mathbf{H} \mathbf{H}^H + \mathbf{I} \sigma^2,
$$

and $\mathbf{w}_n$ corresponds to the $n$th column of $\mathbf{W}$.

It is possible to estimate the distribution of the bits ($b_n \in \{-1, +1\}$) of the elements of $\mathbf{z}$ as a Gaussian random variable, as shown in [9], with a probability density function (PDF) of:

$$
f(z_n|b_n) = \frac{1}{\sqrt{2\pi\sigma_n}} \exp \left[ -\frac{(z_n - m_n)^2}{2\sigma_n^2} \right], \quad n = 1, ..., N,
$$

where $m_n$ and $\sigma_n^2$ are the mean and variance, respectively,

$$
m_n = \frac{\bar{z}_n}{1 + \bar{\gamma}_n} b_n, 
$$

$$
\sigma_n^2 = \frac{\bar{\gamma}_n}{2(1 + \bar{\gamma}_n)^2},
$$

with $\bar{\gamma}_n$ representing the instantaneous SINR of transmit antenna $n$, approximated as below:

$$
\bar{\gamma}_n = \mathbf{h}_n^H \mathbf{R}_n^{-1} \mathbf{h}_n,
$$

where $\mathbf{R}_n$ is the auto-correlation matrix of the interference plus noise.

### III. INTERFERENCE CANCELLATION TECHNIQUES

In this section, we shall review the interference cancellation techniques that are involved in our proposed MF-RO-SIC detector, with an overview of SIC, LLR-based RO and MF, and descriptions of the algorithms required.

**A. Successive Interference Cancellation**

The method of SIC is based upon the theory that if in a system where multiple signals are interfering with each other, if each signal is estimated individually serially, then the interference effects of an estimated signal can be removed from the signals that have yet to be estimated, thus increasing the reliability of estimation of the remaining signals. The linear MMSE filter detection method is commonly used within this process to estimate the symbols in a MIMO received signal vector, the estimate is then quantised appropriately for the process to estimate the symbols in a MIMO received signal.

**Table I**

**Table I: Successive Interference Cancellation Algorithm**

<table>
<thead>
<tr>
<th>Initialization: $y_0 = y$, $H_0 = H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>for $i = 1 \rightarrow N$ do</td>
</tr>
<tr>
<td>$z_i = y_{i-1}$</td>
</tr>
<tr>
<td>$\hat{x}_i = [\bar{z}]_i$</td>
</tr>
<tr>
<td>$y_i = y_{i-1} - \hat{x}_i \mathbf{h}_i$</td>
</tr>
<tr>
<td>$H_i = H_{i-1}(i)$</td>
</tr>
<tr>
<td>end for</td>
</tr>
</tbody>
</table>

$H^{(i)}$ represents $H$ with the $i$th column removed $Q[\cdot]$ represents the quantise function

**Table II**

**Table II: Algorithm used for the RO-SIC process**

When a SIC process cancels the estimated signals can make a significant difference to the overall error rate performance of the detection algorithm, as some signals will have a greater effect of interference than others. If the signal being estimated has a high level of interference, the symbol estimate produced has a greater chance of being inaccurate. If this inaccuracy is then used for cancellation in the algorithm, this may cause the other signals to also be estimated incorrectly, in an effect known as error propagation.

To reduce the impact of this effect, we can order the signals by the greatest associated channel power first, and the weakest last, as this ensures that the signals with a good SINR are estimated first, and are thus less likely to be unreliably estimated. This also means the interferer with the greatest power is cancelled first, improving the SINR of the remaining signals by the greatest amount. This ordering by channel power is known as Ordered SIC (OSIC) or the VBLAST algorithm.

In a quasi-static channel environment, this can be calculated once per packet, and the resultant ordering used for every time instant in that packet.

**B. Log Likelihood Ratio Based Reliability Ordering**

As the well-known VBLAST technique is based on the average channel power per antenna, which does not consider the received signal in a given time instant, it does not take into consideration the per-time instant fluctuations in SINR at the receive antennas.

However, if we consider the LLR at each time instant of transmission, we can derive a dynamic ordering of the SIC process. First, let us consider the LLR of a bit of a received symbol given the estimated detection quantity $z_n$:

$$
L_n = \ln \left( \frac{f(z_n|b_n = +1)}{f(z_n|b_n = -1)} \right).
$$

The magnitude of $L_n$ can be considered a measure of how certain the detector can be that the estimated symbol is correct when the quantisation of $z_n$ is performed, and thus can be considered a measure of the reliability of the estimated bit. The reliability $L_n$ can be approximated if we substitute the Gaussian PDF approximation of $z_n$, from Eq.(7), resulting in:

$$
L_n = 4(1 + \bar{\gamma}_n) |z_n| \equiv (1 + \bar{\gamma}_n) |z_n|.
$$

However, $\bar{\gamma}_n$ requires $\mathbf{R}_n^{-1}$, which may be complex to acquire in the system. In [10], the matrix inversion lemma may be applied to $\mathbf{R}_n^{-1}$, resulting in:

$$
\bar{\gamma}_n = \frac{\mathbf{h}_n^H \mathbf{R}_n^{-1} \mathbf{h}_n}{1 - \mathbf{h}_n^H \mathbf{R}_n^{-1} \mathbf{h}_n},
$$

which only uses $\mathbf{R}_n^{-1}$, which is already obtained for use in the MMSE filter. Substituting this result into Eq.(12) and simplifying gives:

$$
L_n = (1 - \mathbf{h}_n^H \mathbf{R}_n^{-1} \mathbf{h}_n)^{-1} |z_n|.
$$

This reliability $L_n$ can be incorporated into the SIC algorithm, as at every stage of the SIC procedure for each antenna’s signal that has not yet been estimated, the reliability can be calculated, and the signal with the highest reliability measurement will be the next estimated signal. This results in a dynamic ordering that is not predetermined before the SIC is processed, and can change with every time instant.

**Table II** shows the algorithm used for the RO-SIC process.
C. Multiple Feedback Cancellation

The method of MF in a SIC process is based on the idea of reliable and unreliable symbol estimates, and how alternative symbol quantisation decisions can affect the cancellation results. During the SIC cancellation process, the quantised estimated symbol \( \hat{x}_n \), defined as \( \hat{x}_n = Q[z_n] \), where \( Q[] \) is the quantisation function appropriate for the modulation scheme being used in the system, and the quantisation operates by choosing the constellation point with the smallest Euclidean distance to the estimated symbol,

\[
\hat{x}_n = \arg\min_{z \in \mathbb{X}} \|z - c\|. \tag{16}
\]

However, if the estimated symbol is close to the Voronoi boundaries of two or more constellation points, then the estimated symbol could conceivably be attributed to a constellation point that does not have the smallest Euclidean distance from the estimated symbol, as the effects of noise and interference are likely to cause the estimate to cross a Voronoi boundary.

Therefore, an area surrounding the Voronoi boundaries can be described as a scenario in which it is possible that the standard quantisation process may give inaccurate results. And so, a shadowing region on the constellation diagram is created, which defines an area in which an estimated symbol may be considered for alternative quantisation results. If \( z_n \) falls within this region, then the \( C \) alternative constellation points with the smallest Euclidean distances \( (U \subset V \{u_1, ..., u_C\}) \) to \( z_n \) are considered for SIC processing, instead of just the closest. Fig. 1 shows how the shadowing region is defined by a shadowing criterion value \( S \) for QPSK modulation, which defines how far from the constellation axes (which are the Voronoi boundaries for a QPSK constellation) the shadowing region is set.

When these alternative candidates for \( \hat{x}_n \) are considered, the MF algorithm processes the SIC to completion for the \( C \) possible candidates, to produce an estimated symbol vector for all transmitted antennas \( \hat{x}_s, s = 1, ..., S \). Then using the ML rule, the MF technique chooses the \( \hat{x}_s \) that produces the smallest Euclidean distance as described by:

\[
\hat{x}_s = \arg\min_{k=1, ..., C} \|y - \mathbf{H}_s \tilde{x}_k\|^2
\]

The symbol candidate associated with the chosen \( \hat{x}_s \) is then chosen as \( \hat{x}_n \) for that cancellation stage, and the MF-SIC continues from the \( n \)th stage.

IV. PROPOSED MULTIPLE FEEDBACK RELIABILITY ORDERING SUCCESSIVE INTERFERENCE CANCELLATION

Our proposed MF-RO-SIC detector combines the ideas of dynamic ordering within the SIC process, the multiple candidates feature for unreliable cancellation estimates, and overcomes the drawbacks of existing SIC detectors by minimising the extra computational complexity required. A possibility for the combination of the MF-SIC and RO-SIC is to have the MF process taking place after the RO algorithm has decided which signal to estimate for this current iteration of the main SIC for loop. But, as may be noted in the algorithm in Table III, the MF-SIC adds only a small amount of extra complexity over the SIC, as the cancellation filters for each stage \( \mathbf{W}_f \) can be precalculated and reused within the MF process:

\[
\mathbf{w}_i = (\mathbf{H}_i \mathbf{H}_i^H + \mathbf{I}_0^2)^{-1} \mathbf{h}_i, i = 1, ..., N, \tag{18}
\]

but this is only true if the cancellation order is known before the SIC is processed. In the case of the RO-SIC, the ordering dynamically changes with each cancellation stage, and so the MF filters cannot be precalculated for each time instant. For
this reason, the filters would have to be recalculated for each stage in the MF, every time $z_n$ falls within the MF shadowing region, as well as determining the RO. This would involve a large increase in complexity over the base MF-SIC, as each filter calculation requires a matrix inversion.

In order to avoid this large increase in required complexity, we can split the MF-RO-SIC into two sections. Firstly, the RO-SIC can be calculated as in Table II, in order to give a base estimate of $\hat{x}$, but also returns the cancellation order taken by the dynamic ordering for this time instant.

\[
y \rightarrow \text{RO-SIC} \xrightarrow{\text{Cancellation Order}} \text{MF-SIC} \xrightarrow{\hat{x}}
\]

Fig. 2. Structure of piece-wise MF-RO-SIC

This returned ordering can then be used as a predetermined ordering for the MF-SIC process, and so the filters can be precalculated as in Table III, with the new ordering, which means the filters only have to be calculated once per time instant for the MF-SIC, reducing the complexity of the MF-SIC process to the original level. This piece-wise approach is shown in Fig. 2.

This piece-wise approach to the MF-RO-SIC does save complexity over the previous algorithm possibility, but filters are still calculated for the MF-SIC, and in effect the complexity of the RO-SIC and the MF-SIC is additive, roughly doubling the complexity over either individual algorithm.

However, greater reductions in complexity can be achieved by further integrating the two stages together intelligently. The MF-SIC filters $g_i$ can be extracted from each stage of the RO-SIC, by taking and storing the $j$th column of the RO-SIC filter associated with the chosen cancellation, thus these values can be reused for the MF-SIC, so that the filters do not need to be calculated at all for the MF-SIC, reducing the complexity as no extra matrix inversions have to be performed. The MF-SIC filter $g_i$ is given by:

\[
g_i = W_{i,j}, \quad (19)
\]

where $W_{i,j}$ is the $j$th column of the filter $W$ calculated for the $i$th cancellation stage of the RO-SIC. Also, whilst the MF-SIC cannot be directly integrated into the RO-SIC with lower complexity due to the dynamic ordering restriction, the shadow criterion test for each $z_n$ can still be carried out within the RO-SIC process, as below:

\[
|\Re \{z_n\}| < S \text{ and } |\Im \{z_n\}| < S \quad (20)
\]

If no $z_n$ falls within the shadow criterion area during the RO-SIC, then logically the MF-SIC will not give a different $\hat{x}$ than the RO-SIC, as the MF technique will never be performed, and so the MF-SIC stage can be skipped for that time instant, reducing complexity further. Fig. 3 shows the structure of the MF-RO-SIC with these considerations.

V. SIMULATION RESULTS

For the simulation results presented in this paper, a MIMO system was considered with QPSK modulation and with perfect knowledge of the channel state information, and the SIC detectors considered are based upon minimum-mean-square-error (MMSE) filtering.

| Table IV: Multiple Feedback Reliability Ordering Successive Interference Cancellation Algorithm |
| Initialisation: $y_0 = y, H_0 = H, \text{RO,}_0 = \text{HH}^\dagger + \text{L,}^{\mbox{m,}0}$ |
| for $i = 1 \rightarrow N$ do |
| $W_i = \text{RO,}_i H_{i-1}$ |
| $z_i = W_i^H y_i-1$ |
| for $j = 1 \rightarrow (N - i + 1)$ do |
| $L_j = (1 - h_j^H H_j^{-1} h_j)^{-1} |z_j|$ |
| end for |
| $j = \arg \max_j L_j (L_1, ..., L_{N-i+1})$ |
| $g_i = W_i x_j$ |
| if $|\Re \{z_j\}| > |\Im \{z_j\}| < S$ then |
| $m = 1$ |
| end if |
| $y_i = y_{i-1} - \hat{x}_i h_j$ |
| $H_i = H_{j-1} w_j$ |
| $\text{RO,}_i = \text{RO,}_{j-1} - h_j h_j^H$ |
| end for |
| if $m = 1$ then |
| $z_i = W_i^H y_i-1$ |
| if $|\Re \{z_i\}| > |\Im \{z_i\}| > S$ then |
| $\hat{x}_i = z_i$ |
| end if |
| for $k = 1 \rightarrow C$ do |
| $x^k = x$ |
| $y^k = y_{i-1} - u_k h_i$ |
| end for |
| for $l = i \rightarrow N$ do |
| $x_{l}^{k} = z_{l}^{k} = z_{l}^{k}$ |
| $y_{l}^{k} = y_{l-1}^{k} - z_{l}^{k} h_{l}$ |
| end for |
| $k_{\text{opt}} = \arg \min_{k=1,...,C} ||y_0 - H_0 \hat{x}_k||^2$ |
| $\hat{x}_i = u_{k opt}$ |
| $y_i = y_{i-1} - \hat{x}_i h_i$ |
| $H_i = H_{j-1} w_j$ |
| end for |
| end if |

Fig. 4 shows how the proposed MF-RO-SIC detector compares with the VBLAST-SIC, RO-SIC, MF-SIC and the ML detector in terms of BER in a 4x4 MIMO system. The number of candidates for the MF ($C$) is set to 4, with the shadow criterion $S$ set to 0.2. The MF-RO-SIC can be seen to have up to 4dB BER performance over the MF-SIC, just over 4dB over the RO-SIC and up to 8dB of gain over the VBLAST-SIC.

Fig. 5 similarly shows how the proposed MF-RO-SIC detector compares with the VBLAST-SIC, RO-SIC, MF-SIC and the ML detector in terms of BER in an 8x8 MIMO system. The number of candidates for the MF ($C$) is set to 4, with the shadow criterion $S$ set to 0.2. The MF-RO-SIC can be
seen to have up to 4dB BER performance over the MF-SIC, just over 2dB over the RO-SIC and up to 8dB of gain over the VBLAST-SIC. It can be noted that in Fig. 4, the MF-SIC had better performance than the RO-SIC, but in Fig. 5, the RO-SIC outperformed the MF-SIC, suggesting that for larger MIMO systems, the RO in the MF-RO-SIC provides the major contribution to the performance gains of the MF-RO-SIC over the VBLAST SIC, whilst for smaller MIMO systems, the MF provides the greater performance gains.

![Figure 4](image1.png)

**Fig. 4.** 4x4 MIMO with QPSK modulation, $C = 4, S = 0.2$

![Figure 5](image2.png)

**Fig. 5.** 8x8 MIMO with QPSK modulation, $C = 4, S = 0.2$

![Figure 6](image3.png)

**Fig. 6.** 4x4 MIMO with QPSK modulation, $C = 4$, variable $S$

In this paper, a new MF-RO-SIC detector has been proposed, utilising the ideas of dynamic cancellation stage reliability ordering and multiple candidates for estimated symbol decisions, with consideration given to the complexity of the resultant detector and how this can be reduced. Simulation results have shown up to 4dB of gains over previously established SIC detectors and 8dB gains over the VBLAST-SIC, for 4x4 and 8x8 MIMO systems, and the effects on the BER that altering the shadow criterion for the MF process in the MF-RO-SIC.

**VI. CONCLUSIONS**

In this paper, a new MF-RO-SIC detector has been proposed, utilising the ideas of dynamic cancellation stage reliability ordering and multiple candidates for estimated symbol decisions, with consideration given to the complexity of the resultant detector and how this can be reduced. Simulation results have shown up to 4dB of gains over previously established SIC detectors and 8dB gains over the VBLAST-SIC, for 4x4 and 8x8 MIMO systems, and the effects on the BER that altering the shadow criterion for the MF process in the MF-RO-SIC.

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