Joint Beamforming and Transmit Design for the Non-Regenerative MIMO Broadcast Relay Channel

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Abstract—In this paper, we consider a multiple-input multiple-output (MIMO) broadcast relay channel (BRC), in which the communication of a multi-antenna base station (BS) with several multi-antenna mobile stations (MS) is assisted by a fixed half-duplex multi-antenna relay station (RS). Applying dirty paper coding (DPC) at the BS and beamforming at the RS, we jointly optimize the transmit covariance matrices at the BS and the beamforming matrix at the RS by maximizing the system sum rate, which is a nonconvex problem. To solve this problem, we resort to the more tractable sum rate maximization in the dual multiple access relay channel (MARC), which is still a nonconvex difference of convex functions (DC) problem. To solve this problem, we resort to the alternating optimization of the beamforming matrix and the transmit covariance matrices. The resulting covariance matrices for the MARC are then mapped to the desired BRC covariance matrices. The sum rate performance of the proposed algorithm is demonstrated by simulations.

Index Terms—Multiuser MIMO relaying, sum rate maximization, alternating optimization, difference of convex functions.

I. INTRODUCTION

In recent years, multiple-input multiple-output (MIMO) systems have been employed in numerous wireless communication systems, such as point-to-point multiple-antenna communications and cellular multi-user communications [1]. It is well known that multiple antennas at wireless terminals can achieve channel capacity enhancements and robustness against channel fading. Furthermore, with the deployment of fixed relay stations (RS) at the cell edges or severely blocked areas in cellular systems, the coverage area can be extended to enhance the throughput of cell-edge users [2]. Various relay strategies have been studied in the literature. The most prominent strategies are amplify-and-forward (AF) and decode-and-forward (DF). Due to its simplicity, the AF scheme is usually preferred in practice.

In order to exploit the above-mentioned benefits, a promising compound scheme that incorporates MIMO technology into the fixed relay architecture with single antenna users was introduced in [3]. The joint source-relay optimization for an AF-based MIMO broadcast relay channel (BRC) with multi-antenna mobile stations (MSs) was considered in [4]. The maximization of the sum rate to jointly optimize the transmit covariance matrices at the base station (BS) and the beamforming matrix at the RS as in [4] is an intricate task. This is due to the nonconvex nature of the optimization problem, which is a difference of convex functions (DC) programming problem. The iterative algorithm developed in [4] is based on solving the same problem for the more tractable dual multiple access relay channel (MARC) by performing DC iterations and hence is suboptimal. Recently, a polynomial time DC (POTDC) method [5] for the class of DC problems with the optimization over a single variable was developed and its global optimality was proven in [6]. The authors of [7] and [8] have subsequently extended the POTDC algorithm to the optimization over a matrix.

In this paper, we consider a MIMO BRC, where a BS simultaneously transmits to K MSs through a fixed half-duplex RS. All parties are equipped with multiple antennas and perfect CSI is assumed everywhere in the network [3], [4]. We jointly optimize the transmit covariance matrices at the BS and the beamforming matrix at the RS by maximizing the system sum rate. Similarly to [4], we resort to the more tractable sum rate maximization in the dual MARC to solve the original BRC problem. We develop an iterative algorithm, termed alternating matrix POTDC algorithm, based on an alternating optimization of the beamforming matrix and the transmit covariance matrices. The resulting covariance matrices for the MARC are then mapped to the desired BRC covariance matrices. Simulation results demonstrate that the proposed alternating POTDC algorithm outperforms the method in [4].

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. MIMO Broadcast Relay Channel

Consider the MIMO BRC scenario depicted in Fig. 1, where the BS serves K MSs simultaneously with the help of a single RS. We assume that there is no direct link between the BS and the K MSs. The BS, the RS, and the i-th MS are equipped with $M_s$, $M_r$, and $M_i$ antennas, respectively. Moreover, we assume that all the channels are flat block-fading channels, i.e., they are constant over a block length and independent from block to block. Perfect channel state information (CSI) is considered at the BS, RS, and the MSs [3], [4]. The transmission takes place in two phases.

In the first phase, the BS employs dirty paper coding (DPC) and transmits $x = \sum_{i=1}^{K} x_i$ to the RS, where $x_i \in \mathbb{C}^{M_i}$ is the codeword transmitted to the i-th MS with the covariance matrix $\Sigma_i = E(x_i x_i^H)$. Hence, the covariance matrix of $x$ is given by...
\[ \Sigma = \mathbb{E}\{xx^H\} = \sum_{i=1}^{K} \Sigma_i. \]

The received signal at the RS is given by \( y_r = \mathbf{H}x + n_r \), where \( \mathbf{H} \in \mathbb{C}^{M_r \times M_t} \) is the BS-to-RS channel and \( n_r \in \mathbb{C}^{M_r} \) contains the i.i.d. noise samples at the RS with \( n_r \sim \mathcal{CN}(0, \sigma^2_{n_r} \mathbf{I}_{M_t}). \) The transmission power at the BS is constrained by \( \sum_{i=1}^{K} \text{Tr}(\Sigma_i) = P, \) where \( P \) is the maximum allowable transmit power and the subscript B stands for BRC.

In the second transmission phase, the BS processes the received signal \( y_t \) by the beamforming matrix \( \mathbf{D}_B \in \mathbb{C}^{M_t \times M_t} \) and transmits the vector \( x_t = \mathbf{D}_B y_t = \mathbf{D}_B \mathbf{H}_r x_t + \mathbf{D}_B n_t \) to the \( K \) MSs. The received signal at the i-th MS can be expressed as
\[
y_i = G_i \mathbf{D}_B H x_t + \sum_{j<i} G_{ij} \mathbf{D}_B H x_j + \sum_{j>i} G_{ji} \mathbf{D}_B H x_j + G_{ii} \mathbf{D}_B n_t + n_i, \tag{1}
\]
where \( G_i \in \mathbb{C}^{M_i \times M_t} \) is the channel from the RS to the i-th MS and \( n_i \in \mathbb{C}^{M_i} \) is the vector of i.i.d. noise samples at the i-th MS with \( n_i \sim \mathcal{CN}(0, \sigma^2_{n_i} \mathbf{I}_{M_t}). \) The power transmit constraint at the BS is given by \( \text{Tr}(\sum_{i=1}^{K} H \Sigma_i H^H + \sigma^2_{n_i} \mathbf{I}_{M_t}) \leq P, \) where \( P \) is the maximum relay transmit power.

Applying the concept of DPC, the BS chooses codeword \( x_i \) for the i-th MS by taking into account the knowledge of the interference caused by the previously chosen codewords \( x_j \) for \( j < i. \) Thus, the BS pre-cancels the interference term \( \sum_{j=1}^{i-1} G_{ij} \mathbf{D}_B H x_j \) at the i-th MS.

### B. MIMO Multiple Access Relay Channel

Next, we consider the corresponding dual MIMO MARC, which is given by reversing the direction of transmission in the BRC illustrated in Fig. 1. Thus, the MSs transmit data to the BS via the RS. Assuming reciprocal channels, the channel from the i-th MS to the RS and the RS-to-BS channel are given by \( G_i^H \in \mathbb{C}^{M_r \times M_t}, i = 1, \ldots, K, \) and \( H_i \in \mathbb{C}^{M_t \times M_t}, \) respectively. Similarly to the BRC model, the transmission is carried out in two phases. In the first phase, all the terminals transmit simultaneously to the RS. Then, the RS processes its received signal by the beamforming matrix \( \mathbf{D}_B \in \mathbb{C}^{M_t \times M_t} \) and retransmits the amplified signal to the BS in the second phase.

Define \( u \in \mathbb{C}^{M_t} \) as the transmitted codeword from the i-th MS with the covariance matrix \( \mathbf{Q}_i = \mathbb{E}\{uu^H\}. \) Therefore, the received signal at the BS in the second phase is
\[
v = \mathbf{H}^H \mathbf{D}_B \mathbf{Q}_i \mathbf{G}_i \mathbf{u} + \mathbf{H}^H \mathbf{D}_B n_t + n_i, \tag{2}
\]
where \( \mathbf{G}_i = [\mathbf{G}_1, \ldots, \mathbf{G}_K]^H, \) and \( \mathbf{u} = [u_1^T, \ldots, u_K^T]^T \) are the concatenated MS-RS channels and the transmit vectors from all the MSs, respectively. The noise vector \( n_r \) is again the noise vector at the RS and \( n_i \in \mathbb{C}^{M_i} \) contains the i.i.d. noise samples at the BS with \( n_i \sim \mathcal{CN}(0, \sigma^2_{n_i} \mathbf{I}_{M_t}). \) The sum power constraint across the MSs and the transmit power constraint of the RS are given by \( \sum_{i=1}^{K} \text{Tr}(\mathbf{Q}_i) = P_M, \) and \( \text{Tr}(\sum_{i=1}^{K} \mathbf{G}_i^H \mathbf{Q}_i \mathbf{G}_i + \sigma^2_{n_i} \mathbf{I}_{M_t}) \leq P_M, \) respectively, where \( P_M \) and \( P_M \) are the maximum power limits and \( M \) stands for MARC.

### C. Uplink-Downlink Duality and Problem Statement

It was established in [9] that duality holds between the BRC model and its dual MARC model for multuser MIMO AF relaying if the beamforming matrices are given by \( \mathbf{D}_M = \mathbf{D} \) and \( \mathbf{D}_B = \mathbf{D}^H(c \in \mathbb{R}_+) \) or vice versa. As a result, the BRC and the MARC expend the same total power, i.e., \( P_M = P_M + P_M = P_M + P_M \) and achieve the same user rates, i.e., \( R_B = R_M, \) and therefore also the same sum rates \( R_B = R_M. \) The power normalization factor \( c \) is chosen according to the type of power constraints at the source and the RS. In the case of a joint power constraint, \( c \) is given by \( c = 1. \)

In this work, however, we only consider the more realistic case of separate power constraints. Thus, the total source and relay powers can be switched such that \( P_B = P_M, \) and \( P_B = P_M, \) and we have \( c = P_M / \text{Tr}(\sum_{i=1}^{K} \mathbf{G}_i^H \mathbf{Q}_i \mathbf{G}_i + \sigma^2_{n_i} \mathbf{I}_{M_t}) \) \cite{9}.

In the MIMO BRC and for an arbitrarily decreasing encoding order \( \pi(i) = \{K, K-1, \ldots, 1\}, \) i.e., MS_K is encoded first, MS_K-1 second, and MS_1 last, the achievable rate for the i-th MS is given by
\[
R_{B_i} = \frac{1}{2} \log \left| \sum_{i=1}^{K} H \Sigma_i H^H + \sigma^2_{n_i} \mathbf{I}_{M_t} \right| D_B^H \mathbf{G}_i^H + \sigma^2_{n_i} \mathbf{I}_{M_t} \right|.
\]

Thus, the sum rate maximization problem becomes
\[
\begin{align*}
\max_{\Sigma, \mathbf{x}_i \succeq 0, \mathbf{D}_B} & \quad \sum_{i=1}^{K} R_{B_i} \\
\text{s.t.} & \quad \sum_{i=1}^{K} \text{Tr}(\Sigma_i) = P_B, \\
& \quad \text{Tr}(\sum_{i=1}^{K} H \Sigma_i H^H + \sigma^2_{n_i} \mathbf{I}_{M_t}) \leq P_B, \\
& \quad \mathbf{Q}_i = \mathbb{E}\{uu^H\}, \quad u \in \mathbb{C}^{M_t}
\end{align*}
\tag{3}
\]

In the dual MARC, the decoding order must be the reverse of the encoding order in the BRC. Hence, for the decoding order \( \pi(i) = \{1, 2, \ldots, K\}, \) i.e., MS_1 is decoded first, MS_2 second, and MS_K last, the achievable rate for the i-th MS is expressed as
\[
R_{M_i} = \frac{1}{2} \log \left| \sum_{j=1}^{K} G_{ji}^H \mathbf{Q}_j \mathbf{G}_j + \sigma^2_{n_j} \mathbf{I}_{M_t} \right| D_M^H \mathbf{H}^H + \sigma^2_{n_j} \mathbf{I}_{M_t} \right|.
\]

while the sum rate is given by
\[
R_M = \sum_{i=1}^{K} R_{M_i} \tag{4}
\]

Therefore, the sum rate maximization problem can be stated as
\[
\begin{align*}
\max_{\mathbf{Q}_i, \mathbf{Q}_i \succeq 0, \mathbf{D}_M} & \quad R_M \\
\text{s.t.} & \quad \sum_{i=1}^{K} \text{Tr}(\mathbf{Q}_i) = P_M, \\
& \quad \text{Tr}(\sum_{i=1}^{K} G_{ij}^H \mathbf{Q}_j \mathbf{G}_j + \sigma^2_{n_j} \mathbf{I}_{M_t}) \leq P_M, \\
& \quad \mathbf{Q}_i = \mathbb{E}\{uu^H\}, \quad u \in \mathbb{C}^{M_t}
\end{align*}
\tag{5}
\]
III. SUM RATE MAXIMIZATION FOR THE BRC

In this section, we address the sum rate maximization in the BRC by solving the same problem for the MARC followed by a mapping of the obtained covariance matrices.

A. Alternating Matrix POTDC Algorithm for the MARC

In this section, we present the alternating matrix POTDC algorithm to address the MARC problem stated in (5). In order for the duality to hold, we choose the beamforming matrices to be $D_M = D$ and $B_M = cDH$. Furthermore, we define the block diagonal matrix $Q ≜ \text{blkdiag}(Q_1, \ldots, Q_K) \subseteq \mathbb{C}^{(\sum_i M_i) \times (\sum_i M_i)}$. Consequently, the objective function of (5) can be written as

$$R_M(Q, D) = \log \left| \frac{\left| H^H D \right|^2 \left| \sum_{i=1}^K \sigma_i^2 G_i \right|^2}{\left| H^H D \sum_{i=1}^K \sigma_i^2 I_{M_i} \right|^2} + \frac{\sigma_n^2}{\sum_{i=1}^K \sigma_i^2 I_{M_i}} \right| \tag{6}$$

where we have omitted the factor of $1/2$. Hence, (5) becomes

$$\max_{Q,D} \quad R_M(Q, D) \tag{7}$$

s.t. $\text{Tr} \{ Q \} = \text{Tr} \{ I \}, \quad Q \succeq 0,$

$$\text{Tr} \{ D (G^H Q + \sigma_n^2 I_{M_i}) D^H \} = \text{Tr} \{ P_M \}. \tag{8}$$

Next, we make use of the capacity-optimal result for single-user MIMO AF relay channels in [10]. Let the SVD of the channel matrices $H$ and $G$ be given by $H = U_H \Sigma_H V_H^H \in \mathbb{C}^{M \times M}$ and $G = U_G \Sigma_G V_G^H \in \mathbb{C}^{N \times M}$, respectively. According to [10], the beamforming matrix $D$ is constrained to possess the structure

$$D = U_H \Sigma_D V_D^H, \tag{9}$$

where $\Sigma_D \in \mathbb{C}^{N \times M}$ is a diagonal matrix. Hence, the matrix $D$ is chosen as a matched filter to the left and right singular vectors of the forward and the backward channels. Using (8), the objective function (6) can be expressed as

$$R_M(Q, \Lambda_D) = \log \left| \frac{V_D^H (G^H Q G + \sigma_n^2 I_{M_i}) V_D \Lambda_D + \sigma_n^2 I_{M_i}}{\Lambda_D + \sigma_n^2 I_{M_i}} \right|, \tag{10}$$

where $\Lambda_D = \Sigma_D^2 \geq 0$, $\Lambda_0 = \Sigma_0 \Sigma_0^H$, and we have used the matrix identity $|AB| = |A||B|$ for arbitrary matrices $A \in \mathbb{C}^{M \times N}$ and $B \in \mathbb{C}^{N \times M}$. Note that (7) with the objective function (9) is still a nonconvex DC problem in both $Q$ and $\Lambda_D$, but it is separable in these variables. This gives rise to an alternating optimization of (7), where we first optimize with respect to $\Lambda_D$ for a fixed value of $Q$, applying an extended version of the POTDC algorithm [5]; then, we fix the obtained value of $\Lambda_D$ and further optimize with respect to $Q$.

Specifically, we alternately solve the following two optimization problems to obtain $\Lambda_D^{(n)}$ and $Q^{(n)}$ at the $n$-th iteration, $n = 1, 2, \ldots$, until convergence:

$$\Lambda_D^{(n)} = \arg \max_{\Lambda_D} \log \left| V_D^H (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D + \sigma_n^2 I_{M_i} \right| - \log \left| \Lambda_0 \Lambda_D + \sigma_n^2 I_{M_i} \right| \tag{11}$$

s.t. $\text{Tr} \{ V_D^H G_i (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D \} = \text{Tr} \{ P_M \}$, $\Lambda_D \succeq 0$, $Q^{(n)} = \arg \max_{Q} \log \left| V_D^H (G^H Q G + \sigma_n^2 I_{M_i}) V_D \Lambda_D^{(n-1)} + \sigma_n^2 I_{M_i} \right| \tag{12}$$

s.t. $\text{Tr} \{ Q \} = \text{Tr} \{ I \}, \quad Q \succeq 0,$

$$\text{Tr} \{ V_D^H G_i (G^H Q G + \sigma_n^2 I_{M_i}) V_D \Lambda_D^{(n-1)} \} = \text{Tr} \{ P_M \}. \tag{13}$$

In the first step, we address the problem (10) via the POTDC algorithm [5], which has been designed for DC problems with functions of a single variable in the nonconcave term. As this is, however, not the case in (10), we first modify (10) in order for the POTDC algorithm to be applicable. To this end, we utilize the property that the determinant of a diagonal matrix $A = \sum_{i=1}^K \lambda_i A_i$ is equal to the product of its diagonal entries, i.e., $|A| = \prod_{i=1}^K a_i$. Applying this property to (10), we obtain

$$\max_{\Lambda_D} \log \left| V_D^H (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D + \sigma_n^2 I_{M_i} \right| - \sum_{i=1}^K \log (\lambda_i \lambda_i + \sigma_n^2) \tag{14}$$

s.t. $\text{Tr} \{ V_D^H G_i (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D \} = \text{Tr} \{ P_M \}$, $\Lambda_D \succeq 0$. \hspace{1cm} (15)

All the terms in (12) are concave or linear in $\Lambda_D$, except for the terms $- \sum_{i=1}^K \log (\lambda_i \lambda_i + \sigma_n^2)$, $i = 1, \ldots, K$, which are convex and each of which now depends only on a single variable. Hence, we apply the POTDC algorithm [5] and iteratively handle these convex constraints in terms of their linear approximation around suitably selected points. The linear approximation of the convex part of (12) around the point $\lambda_i = 0$ is $\text{Tr} \{ V_D^H G_i (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D \} = \text{Tr} \{ P_M \}$, $\Lambda_D \succeq 0$, $Q^{(n)} = \arg \max_{Q} \log (\lambda_i \lambda_i + \sigma_n^2)$.

In order to solve (12), we use (13) and perform iterations using interior-point methods over the following problem:

$$\max_{\Lambda_D} \log \left| V_D^H (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D + \sigma_n^2 I_{M_i} \right| - \sum_{i=1}^K (\lambda_i - \lambda_i) \log \lambda_i \tag{16}$$

s.t. $\text{Tr} \{ V_D^H G_i (G^H Q^{(n-1)} G + \sigma_n^2 I_{M_i}) V_D \Lambda_D \} = \text{Tr} \{ P_M \}$, $\Lambda_D \succeq 0$. \hspace{1cm} (17)

Note that we have omitted the first term of the approximation (13) in problem (14), which is a constant. The obtained POTDC algorithm guarantees the convergence to a Karush-Kuhn-Tucker (KKT) point of the problem (12). Furthermore, the POTDC iterations ensure a non-decreasing sequence of the objective values over the iterations [5].

In the second step of the alternating optimization, we solve problem (11), which is convex in $Q$ for the fixed value of $\Lambda_D$ and can be straightforwardly solved using standard interior-point methods. We summarize the above-developed alternating matrix POTDC optimization procedure for solving the MIMO MARC problem (5) in Algorithm 1. Both optimization steps in Algorithm 1 result in non-decreasing objective values. A more detailed convergence and optimality analysis is the concern of future work.

B. Mapping from MARC to BRC Covariance Matrices

In the previous section, we have addressed the sum rate maximization in the BRC by solving its dual MARC problem. We here provide the mapping of the resulting MARC covariance matrices $Q_i$ to the desired BRC covariances $\Sigma_{ii}$, $i = 1, \ldots, K$. To this end, we define

$$B_i = G_i D_M \left( \sum_{j=1}^{i-1} H \Sigma_j D_j H^H + \sigma_n^2 I_{M_i} \right) D_M^H G_i^H + \sigma_n^2 I_{M_i}, \hspace{1cm} (18)$$

$$M_i = H_i D_M \left( \sum_{j=i+1}^K G_j G_j^H + \sigma_n^2 I_{M_i} \right) D_M^H H + \sigma_n^2 I_{M_i}, \hspace{1cm} (19)$$
Algorithm 1: The alternating matrix POTDC algorithm for solving the MARC problem (5)

1: Initialize $n = 1, \epsilon_1 > 0, \epsilon_2 > 0, Q(0) = \frac{P}{\sigma^2} I_{KM}$; 
2: while $|Q_{n}(i) - Q_{n-1}(i)| > \epsilon_1$ do
3: Set $l = 1$;
4: while $|P_{M}(i) - P_{M}(i-1)| > \epsilon_2$ do
5: Solve (14) to obtain $Q^{(l)}$;
6: $\Lambda_{D}^{(l)} = \Lambda_{D}^{(l-1)}$;
7: $l = l + 1$;
8: end while
9: $\Lambda_{D}^{(n)} = \Lambda_{D}^{(n-1)}$;
10: Solve (11) to obtain $Q^{(n)}$;
11: $n = n + 1$;
12: end while
13: Compute $D^{*} = U_{n}^{H} \Lambda_{D}^{1/2} V_{n}^{H}$;
14: Solve (7) to obtain $Q^{*}$;
15: Output: $Q^{*}, D^{*}$.

as the respective interference-plus-noise covariance matrices seen by the $i$-th MS in the BRC and the dual MARC. Then, the $\Sigma_{i}$ can be generated from the $Q_{i}$ as

$$
\Sigma_{i} = \frac{1}{\epsilon_{2}}M_{i}^{-1/2}U_{i}V_{i}^{H}B_{i}^{1/2}Q_{i}B_{i}^{1/2}V_{i}^{H}U_{i}^{H}M_{i}^{-1/2},
$$

where $M_{i}^{-1/2}H_{i}^{H}DG_{i}^{H}B_{i}^{-1/2} = U_{i}^{H}A_{i}^{H}V_{i}$ is the SVD of the effective channel matrix experienced by the $i$-th MS. Note that $B_{i} = G_{i}D_{i}^{H}G_{i}^{H} + \sigma_{n_{i}}^{2}I_{M_{i}}$ and $M_{K} = H_{i}^{H}D_{i}^{H}H + \sigma_{n_{i}}^{2}I_{M_{i}}$ as $M_{i}$ in the BRC and $M_{K}$ see no interference from the other users. Moreover, $\Sigma_{i}$ only depends on $\Sigma_{j}, j < i$ and thus, the $\Sigma_{i}$ can be computed sequentially from the $Q_{i}$ in an ascending order.

IV. SIMULATION RESULTS

In this section, we present simulation results to demonstrate the performance of the proposed alternating matrix POTDC algorithm “AM-POTDC” for solving the sum rate maximization problem for the MIMO BRC. For comparison purposes, we include the recently proposed method “DC-Iter” in [4] into our evaluation. In the simulations, the channels $H$ and $G_{i}, i = 1, \ldots, K$, are randomly generated and drawn from an i.i.d. complex Gaussian distribution with zero mean and unit variance. Moreover, we assume that the $K$ MSs are equipped with the same number of antennas, i.e., $M_{i} = M$. For the proposed algorithm and the method from [4], the initialization $Q^{(0)} = \frac{P}{\sigma^2} I_{KM}$ is chosen such that equal power is allocated across all MSs. The thresholds $\epsilon_1$ and $\epsilon_2$ are both set to $10^{-5}$ and the results are obtained by averaging over 1000 independent Monte Carlo trials.

In Fig. 2, we illustrate the sum rate as a function of the signal-to-noise ratio (SNR). We have $M_{t} = M_{i} = 5$, and $K = 2$ users with $M = 5$. The transmit power limits are given by $P_{BH} = P_{BH} = P_{BH} = P_{BH} = 1$. It can be seen that the proposed AM-POTDC algorithm outperforms the DC-Iter method and provides a higher sum rate.

V. CONCLUSION

In this paper, we have considered the nonconvex sum rate maximization problem for the MIMO broadcast relay channel (BRC). In this scenario, a multi-antenna base station communicates with several multi-antenna mobile stations with the help of a fixed half-duplex multi-antenna relay station. We jointly optimize the transmit covariance matrices at the BS and the beamforming matrix at the RS by maximizing the sum rate. To solve this problem, we resort to the more tractable sum rate maximization in the dual multiple access relay channel (MARC), which is still a nonconvex difference of convex functions (DC) programming problem. We have developed the alternating matrix POTDC algorithm, which is based on an alternating optimization of the beamforming matrix and the transmit covariance matrices. The resulting covariance matrices for the MARC are then mapped to the desired BRC covariance matrices. Simulation results have demonstrated the superior performance of the proposed alternating matrix POTDC algorithm compared to the method in [4].

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Fig. 2. Sum rate versus the SNR for $M_{t} = M_{i} = 5, K = 2$ users with $M = 5$ and $P_{BH} = P_{BH} = P_{BH} = P_{BH} = 1$. The alternating matrix POTDC algorithm compared to the method in [4].

Please note that the algorithm for solving the MARC problem is given in Algorithm 1. The equations and the figure are related to the simulation results and the conclusions drawn from them.