Power Allocation/Beamforming for DF MIMO Two-Way Relaying: Relay and Network Optimization

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Abstract—The problem of sum-rate maximization with minimum power consumption is studied for a decode-and-forward (DF) multiple-input multiple-output (MIMO) two-way relaying system consisting of two sources and one relay. Two scenarios are investigated. In the first scenario, the relay optimizes its own power allocation/beamforming strategy given that the strategies of the sources maximize the sum-rate of the multiple-access channel (MAC) phase. In the second scenario, the relay and the sources jointly optimize their power allocation/beamforming strategies over both the MAC and broadcasting (BC) phases. The considered problem of sum-rate maximization with minimum power consumption is shown to be nonconvex in both scenarios. For the first scenario, an algorithm is proposed to find the optimal strategy of the relay. For the second scenario, the sources and the relay find their strategies either through transferring the original nonconvex problem into corresponding convex problems or using a proposed low-complexity algorithm. Simulation results demonstrate the performance of proposed algorithms.

I. INTRODUCTION

Two-way relaying has recently attracted significant interests [1]-[5]. A typical two-way relay network involves two sources which need to exchange information with each other via one relay. Exploiting bi-directional communication, two-way relaying achieves higher spectral efficiency as compared to one-way relaying.

The performance of two-way relaying depends on the transmission strategies of both the sources and the relay. Power allocation and related problems, e.g., beamforming, have been one of the research interests of two-way relaying. In [2], the optimal power allocation for two-way relaying is studied under an equal rate constraint for both amplify-and-forward (AF) and decode-and-forward (DF) relaying. While this work assumes single antenna at both the sources and the relay, the case with multiple antennas is investigated in [3], [4]. Assuming that the number of antennas at the relay and the sources are the same, the problem of joint sum-rate maximization and transmission power minimization for AF MIMO two-way relaying is investigated in [3]. The optimal power allocation of the relay for MIMO DF two-way relaying considering fairness is derived in [4] assuming that the information rate on each direction is dominated by the rate of the transmission from the relay to the sources. The achievable rate region of MIMO DF two-way relaying is characterized in [5]. The problem of sum-rate maximization with transmission power minimization for MIMO DF two-way relaying is considered in this work. We show that achieving the maximum sum-rate in the MIMO DF two-way relaying does not necessarily require the consumption of all the available power of the relay or sources. Therefore, we optimize the power allocation/beamforming strategies in MIMO DF two-way relaying so that the considered network can achieve both high spectral efficiency, in terms of sum-rate, and high power efficiency at the same time. The optimal strategies are found in two scenarios, i.e., a relay optimization scenario and a network optimization scenario. The former scenario features low overhead while the network-level optimal solution is found in the latter scenario.

II. SYSTEM MODEL

Consider a two-way relay system with two sources and one relay, where source $i$ ($i = 1, 2$) and the relay have $n_i$ and $n_r$ antennas, respectively. In the first phase (MAC phase), source $i$ transmits the signal $W_i s_i$ to the relay, where $W_i$ is the beamforming matrix of source $i$ and $s_i$ is the information symbol vector of source $i$ satisfying $E[s_is_i^H] = I$ and $E[s_is_i^H] = 0$ with $E[\cdot]$ representing expectation and $\{\cdot\}^H$ standing for conjugate transpose. The channels from source $i$ to the relay and from the relay to source $i$ are denoted as $H_{ir}$ and $H_{ri}$, respectively. It is assumed that the relay has all channel information. The received signal at the relay in the MAC phase is

$$y_r = H_{1r} W_1 s_1 + H_{2r} W_2 s_2 + n_r$$

where $n_r$ is the noise at the relay with covariance $\sigma_n^2 I$. Source $i$ has a power budget limit formulated as $\text{Tr}(W_i W_i^H) \leq P_i$. The sum-rate of the MAC phase is bounded by [7]

$$R_{\text{mac}}(W_1, W_2) = \frac{1}{2} \log \left| 1 + \frac{1}{\sigma_n^2} (H_{1r} W_1 W_1^H H_{1r}^H + H_{2r} W_2 W_2^H H_{2r}^H) \right|$$

In the BC phase, the relay decodes $s_i$ from the received signal, re-encodes messages using superposition coding and transmits

$1$It is assumed as default throughout the paper that the user index $i$ and $j$ satisfy $i \neq j$ when they appear in the same expression/context.
where $T_{ri}$ is the relay beamforming matrix of size $n_r \times n_r$ for relaying the signal from source node $j$ to $i$. The relay has the power limit $\text{Tr}(T_{r1}^H T_{r1} + T_{r2}^H T_{r2}) \leq P_r$. Note that the relay needs to know $W_i$ in order to decode $s_i$. Such information can be obtained by the relay by either using channel knowledge or using feedback. It is not difficult for the relay to calculate the optimal beamforming for both of the sources since it has all channel information. Moreover, in order to achieve sum-rate optimization with minimum power consumption on the network level in a two-way relaying system, it can be necessary for the relay to coordinate transmission strategies of the sources through informing the sources of their optimal beamforming matrices. Such coordination is required especially when the sources do not have sufficient processing capability or necessary channel information for deriving their optimal strategies. Similarly, $T_{rj}$ should be available at source $i$ for self-interference cancelation.

The received signal at source $i$ is

$$y_i' = H_{ri}x_r + n_i$$

where $n_i$ is the noise at source $i$ with covariance $\sigma_i^2 I$. With the knowledge of $H_{ri}$ and $T_{rj}$, source $i$ subtracts the self-interference $H_{ri}T_{rj}s_j$ from the received signal and the equivalent received signal at source $i$ is

$$y_i = H_{ri}T_{rj}s_j + n_i.$$  

The effective information rate for the communication from the relay to source $i$ in this phase can be written as [1]

$$R^{bc}_{ri}(T_{ri}) = \frac{1}{2} \min \{ \hat{R}_{ri}(T_{ri}), \tilde{R}_{jr}(W_j) \}$$

where

$$\hat{R}_{ri}(T_{ri}) = \log |\mathbf{I} + (H_{ri}T_{ri}^H H_{ri}) (\sigma_i^2)^{-1}|$$

$$\tilde{R}_{jr}(W_j) = \log |\mathbf{I} + (H_{jr} W_j W_j^H H_{jr}) (\sigma_j^2)^{-1}|.$$  

The sum-rate in the BC phase is then

$$R^{bc}(T_{r1}, T_{r2}) = R^{bc}_{r1}(T_{r1}) + R^{bc}_{r2}(T_{r2}).$$

The sum-rate for the communication over both phases for the considered DF two-way relaying is then

$$R^{tw} = \min \{ R^{mac}(W_1, W_2), R^{bc}(T_{r1}, T_{r2}) \}.$$  

In the following two sections, the problem of sum-rate optimization with transmission power minimization is considered first for the relay only and then for the network of the two sources and the relay.

### III. Relay Optimization

In the scenario of relay optimization in this section, the relay does not coordinate the sources. Instead it aims at optimizing its own power allocation/beamforming strategy to achieve sum-rate maximization in the BC phase using minimum power.

The sources in this scenario find their beamforming matrices by solving the following convex problem

$$\max_{(W_1, W_2)} R^{mac}(W_1, W_2)$$  

s.t.  

$$\text{Tr}(W_i W_i^H) \leq P_s, \forall i$$

which is a basic power allocation problem on MAC studied in [6]. Denote the solution of the above problem as $W_1^0, W_2^0$.

For the sum-rate optimization, the relay needs to solve the following problem

$$\max_{\{T_{r1}, T_{r2}\}} \min \{ R^{mac}(W_1^0, W_2^0), R^{bc}(T_{r1}, T_{r2}) \}$$

s.t.  

$$\text{Tr}(T_{r1}^H T_{r1} + T_{r2}^H T_{r2}) \leq P_r.$$

The problem (12a)-(12b) is convex. However, in order to find the pair $\{T_{r1}, T_{r2}\}$ with minimum $\text{Tr}(T_{r1}^H T_{r1} + T_{r2}^H T_{r2})$ among all possible $\{T_{r1}, T_{r2}\}$ that achieve the same maximum of the objective function (12a), extra constraints need to be taken into account including the following two$^2$

$$\tilde{R}_{r1}(T_{r1}) \leq \tilde{R}_{jr}(W_j^0), \forall i$$

$$\tilde{R}_{r1}(T_{r1}) + \tilde{R}_{r2}(T_{r2}) \leq R^{mac}(W_1^0, W_2^0).$$

Considering the above constraints, the problem already becomes nonconvex. In order to find the optimal power allocation/beamforming strategy, we develop an efficient algorithm based on waterfilling. Note that waterfilling cannot be directly applied here due to the fact that the relay has to distribute power for the communications on two directions and that $R^{bc}_{ri}(T_{ri})$ is the minimum of two items as shown in (6).

Denote the rank of $H_{ri}$ as $r_{ri}$ and the singular value decomposition (SVD) of $H_{ri}$ as $U_{ri} \Omega_{ri} V_{ri}^H$. Assume that the first $r_{ri}$ diagonal elements of $\Omega_{ri}$ are non-zero, sorted in the descending order and denoted as $\omega_{ri}(1), \ldots, \omega_{ri}(r_{ri})$ while the last $n_r - r_{ri}$ diagonal elements are zeros. Then the algorithm for relay optimization is summarized as in Table I, where the rates $R^{mac}(W_1^0, W_2^0), \tilde{R}_{r1}(T_{r1})$ and $\tilde{R}_{jr}(W_j^0)$ are briefly denoted as $R^{mac}, \tilde{R}_{r1}$ and $\tilde{R}_{jr}$, respectively.

**Theorem 1:** The algorithm in Table I achieves optimal relay power allocation. Based on this power allocation, the following beamformers

$$T_{ri} = V_{ri} P_{ri}, \forall i$$

where $P_{ri}$ is a diagonal matrix of size $n_r \times n_r$ given as

$$P_{ri} = \begin{bmatrix} \sqrt{p_{ri}(1)} & \cdots & \sqrt{p_{ri}(r_{ri})} \\ & \cdots & \\ & \sqrt{p_{ri}(r_{ri})} & 0 \end{bmatrix}, \forall i$$

are the optimal solution of the problem of sum-rate maximization with minimum power consumption in the relay optimization scenario.

$^2$The constraints (13a) and (13b) are necessary constraints presented here only to show that the considered problem is nonconvex.
TABLE I  
The Algorithm for Relay Optimization.

1. Initial watering: allocate $P_r$ on $[\omega_r(k)]^2, \forall k \in \{1, \ldots, r_j\}$, $i = 1, 2$ using watering. Denote the initial water level as $\frac{1}{\lambda_i}$. The power allocated to $[\omega_r(k)]^2$ is then $p_r(k) = (1 - \frac{\sigma^2}{|\omega_r(k)|^2})^+$. Set $M_j^i = \{k | p_r(k) > 0\}, i = 1, 2$.
2. Check if $\sum_{M_j^i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) \leq \tilde{R}_{ir}$ for both $i = 1, 2$. If yes, output $p_r(k), \forall k \in \{1, \ldots, r_j\}, i = 1, 2$ and proceed to Step 6. If there exists $i$ such that $\sum_{M_j^i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) > \tilde{R}_{jr}$, denote the set of such $i$ as $I$ and proceed.
3. For $i \in I$, search for $\lambda_i$ ($\lambda_i > \lambda_0$) such that $[M_j^i]_j \log \lambda_i = \sum_{\lambda_i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) - \tilde{R}_{jr}$, where $p_r(k) = (1 - \frac{\sigma^2}{|\omega_r(k)|^2})^+$, $M_j^i = \{k | p_r(k) > 0\}$ and $[M_j^i]_j$ is the cardinality of set $M_j^i$. Set $\tilde{R}_{ir} = \sum_{M_j^i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2})$. Check if $j \in I, j \neq i$. If yes, proceed to Step 4. Otherwise, proceed to Step 5.
4. Find $\lambda_j$ ($\lambda_j > \lambda_0$) such that $[M_j^j]_j \log \lambda_j = \sum_{M_j^j} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) - \tilde{R}_{jr}$, where $p_r(k) = \left(\frac{\lambda_j}{\lambda_j - \frac{\sigma^2}{|\omega_r(k)|^2}}\right)^+$ and $M_j^j = \{k | p_r(k) > 0\}$. Set $\tilde{R}_{ij} = \sum_{M_j^j} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2})$. Proceed to Step 6.
5. Calculate $P_r' = P_r - \sum_{j} p_r(k)$. Allocate $P_r'$ on $[\omega_r(k)]^2, \forall k \in \{1, \ldots, r_j\}$ via watering. Obtain water level $\frac{1}{\lambda_j}$. Calculate $p_r(k) = \left(\frac{\lambda_j}{\lambda_j - \frac{\sigma^2}{|\omega_r(k)|^2}}\right)^+$ and $M_j^j = \{k | p_r(k) > 0\}$. Set $\tilde{R}_{ir} = \sum_{M_j^j} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2})$. If $\tilde{R}_{ij} \leq \tilde{R}_{ir}$, proceed to Step 6. Otherwise, go to Step 4.
6. Check if $\tilde{R}_{ir} + \tilde{R}_{ij} \leq R_{mac}$. If yes, output $p_r(k), \forall k, \forall i$ and break. Otherwise, find out $\lambda_m$ such that $\sum_{i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) = 1^+ = R_{mac}$. If $\lambda_i < \lambda_0$ and $\lambda_j < \lambda_m$, set $\lambda_i = \lambda_j = \lambda_m$. If $\lambda_i > \lambda_m$ and $\lambda_j < \lambda_m$, update $\lambda_i, p_r(k), \forall k, \forall M_j^j$ such that $[M_j^j]_j \log \lambda_j = \sum_{M_j^j} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}) - R_{mac} + \tilde{R}_{ir}$. Update $p_r(k), \forall k$ and/or $p_r(k), \forall k$ according to new $\lambda_i$ and/or $\lambda_j$.

All proofs in this paper are omitted due to space limitation and will be available in the corresponding journal paper [8].

The algorithm for relay optimization can be briefly understood as follows. Step 1 obtains initial power allocation $P_{r1}, P_{r2}$ and corresponding BC phase rates $\tilde{R}_{r1}(T_{r1})$ and $\tilde{R}_{r2}(T_{r2})$. The beamforming matrices $T_{r1}^{01}, T_{r2}^{02}$ maximize $\tilde{R}_{r1}(T_{r1}) + \tilde{R}_{r2}(T_{r2})$ among all possible $T_{r1}$ and $T_{r2}$ subject to the relay’s power’s constraint. Step 2 checks if $\tilde{R}_{r1}(T_{r1})$ in (6) is upper-bounded by $\tilde{R}_{s1}(W_{s1}^0), \forall i$. If it is true, i.e., $\tilde{R}_{r1}(T_{r1}) > \tilde{R}_{s1}(W_{s1}^0)$, the relay reduces its power allocated for relaying the signal from source $i$ to source $j$ such that $\tilde{R}_{r1}(T_{r1}) = \tilde{R}_{s1}(W_{s1}^0)$ in Step 3. It can be proved that $\tilde{R}_{r1}(T_{r1}) = \tilde{R}_{s1}(W_{s1}^0)$ at the point when $[M_j^i]_j \log \lambda_j = \sum_{M_j^i} \log (\frac{|\omega_r(k)|^2}{\sigma^2}) - \tilde{R}_{ir}$. If $\tilde{R}_{r1}(T_{r1})$ is reduced in Step 3, more power is available for relaying the signal from source $j$ to source $i$. In this case, if $\tilde{R}_{r1}(T_{r1}) > \tilde{R}_{s1}(W_{s1}^0)$, the relay reduces its power allocated for relaying the signal from source $j$ to source $i$ until $\tilde{R}_{r1}(T_{r1}) = \tilde{R}_{s1}(W_{s1}^0)$ in Step 4. Otherwise, $\tilde{R}_{r1}(T_{r1}) < \tilde{R}_{s1}(W_{s1}^0)$ and more power should be allocated for relaying the signal from source $j$ to source $i$ until the power limit is reached or until $\tilde{R}_{r1}(T_{r1}) = \tilde{R}_{r1}(W_{s1}^0)$, which is implemented in Step 5. Step 6 makes sure that constraint (13b) is satisfied.

From the proposed algorithm and the above description, it can be seen that the algorithm in Table I finds the optimal solution of the considered relay optimization problem at most six steps without iterations.

IV. NETWORK OPTIMIZATION

In this section, the relay and the sources jointly optimize their power allocation/beamforming strategies to achieve sum-rate maximization with minimum power consumption for the two-way relaying over two phases.

The sum-rate maximization part can be formulated as the following optimization problem

$$\max \{W_1, W_2\} \min \{R_{mac}(W_1, W_2), R_{bc}(T_{r1}, T_{r2})\} \quad (16a)$$

$$\text{s.t.} \quad \text{Tr}[W_1 W_2^H] \leq P_r, \forall i \quad (16b)$$

$$\text{Tr}[T_{r1} T_{r1}^H + T_{r2} T_{r2}^H] \leq P_r. \quad (16c)$$

The above problem is a convex problem. Similar to the scenario of relay optimization, extra constraints are introduced if transmission power minimization needs to be taken into account. For network optimization, these constraints include

$$R_{mac}(W_1, W_2) = R_{bc}(T_{r1}, T_{r2}) \quad (17a)$$

$$\tilde{R}_{r1}(T_{r1}) \leq \tilde{R}_{s1}(W_{s1}), \forall i. \quad (17b)$$

With the above constraints, the problem becomes nonconvex. Since the optimal $T_{r1}, T_{r2}$ and $W_1, W_2$ depend on each other, solving the considered problem of sum-rate maximization with minimum power consumption for the network optimization scenario generally involves iterative optimization over $T_{r1}, T_{r2}, W_1$, and $W_2$. Such process can be inefficient. Next we show that the optimization of $T_{r1}, T_{r2}, W_1$, and $W_2$ can be decoupled for some cases. Correspondingly, the original problem can be simplified to different convex problems.

In order to decouple the optimization variables of the original problem, we first use an initial power allocation. Obtain $\lambda_1, p_r(k)$ and $M_j^i, \forall i$ via the watering procedure as described in Step 1 of Table I. Denote $T_{r1}$ as the corresponding beamforming matrices obtained from $p_r(k), \forall k$ using (14) and (15), and define

$$\tilde{R}_{r1}(T_{r1}) = \sum_{M_j^i} \log (1 + \frac{p_r(k)|\omega_r(k)|^2}{\sigma^2}), \forall i. \quad (18)$$

Use $\tilde{R}_{r1}(T_{r1}), \forall i$ as the initial BC phase rates. Denote $R_{bc}(T_{r1}, T_{r2}) = \tilde{R}_{r1}(T_{r1}) + \tilde{R}_{r2}(T_{r2})$. Based on the comparison of $R_{mac}(W_1, W_2)$ and $R_{bc}(T_{r1}, T_{r2})$, the original problem is considered in the following two cases, each of which has several subcases.
A. The case when \( R_{\text{mac}}^m(W_1^0, W_2^0) \geq R_{\text{bc}}(T_{r1}^0, T_{r2}^0) \)

This case corresponds to the situation when the maximum achievable sum-rate of the MAC phase is larger than or equal to the maximum achievable sum-rate of the BC phase. There are three subcases in this case.

Subcase I: \( \tilde{R}_{r1}(T_{r1}^0) \leq \tilde{R}_{r2}(W_2^0) \) and \( \tilde{R}_{r2}(T_{r2}^0) \leq \tilde{R}_{r1}(W_1^0) \). The sources find their optimal strategies from solving the convex optimization problem

\[
\min_{\{W_1, W_2\}} \text{Tr}(W_1W_1^H + W_2W_2^H) \\
\text{s.t.} \quad R_{\text{mac}}(W_1, W_2) \geq \tilde{R}_{r1}(T_{r1}^0) + \tilde{R}_{r2}(T_{r2}^0) \\
\tilde{R}_{r1}(W_1) \geq \tilde{R}_{r2}(T_{r2}^0) \\
\tilde{R}_{r2}(W_2) \geq \tilde{R}_{r1}(T_{r1}^0) \\
\text{Tr}(W_iW_i^H) \leq P_i, \forall i.
\]

(19a)

(19b)

(19c)

(19d)

(19e)

The optimal strategy of the relay, which is to use the beamformers \( T_{r1}^0 \) and \( T_{r2}^0 \), consumes all its available power.

Subcase II: \( \tilde{R}_{r1}(T_{r1}^0) > \tilde{R}_{r2}(W_2^0) \), \( \tilde{R}_{r2}(T_{r2}^0) \leq \tilde{R}_{r1}(W_1^0) \), and there exists \( P_1' < P_1 \) and \( P_2' = P_2 \) such that the optimal solution, denoted as \( \{W_1', W_2'\} \), to the problem (11a)-(11b) with \( P_1 \) and \( P_2 \) substituted by \( P_1' \) and \( P_2' \) satisfies \( R_{\text{mac}}(W_1', W_2') = R_{\text{bc}}(T_{r1}^0, T_{r2}^0) \) and \( \tilde{R}_{r1}(W_1') \geq \tilde{R}_{r1}(T_{r1}^0) \).

Theorem 2: In subcase II, the optimal solution to the problem

\[
\max_{\{W_1, W_2\}} \tilde{R}_{r1}(W_i) \\
\text{s.t.} \quad \tilde{R}_{r2}(W_j) \geq \tilde{R}_{r1}(T_{r1}^0) \quad \text{Tr}(W_iW_i^H) \leq P_i, \forall i
\]

(20a)

(20b)

(20c)

denoted as \( W_i^* \), always satisfies \( \tilde{R}_{r1}(W_i^*) > \tilde{R}_{r1}(T_{r1}^0) \).

Based on Theorem 2, it can be seen that the problem (19a)-(19e) is still feasible in subcase II. Therefore, the optimal strategies of the sources and the relay in subcase II are the same as those given in subcase I.

Subcase III: \( \tilde{R}_{r1}(T_{r1}^0) > \tilde{R}_{r2}(W_2^0) \), \( \tilde{R}_{r2}(T_{r2}^0) \leq \tilde{R}_{r1}(W_1^0) \), and there does not exist \( P_1' < P_1 \) and \( P_2' = P_2 \) such that the optimal solution, denoted as \( \{W_1', W_2'\} \), to the problem (11a)-(11b) with \( P_1 \) and \( P_2 \) substituted by \( P_1' \) and \( P_2' \) satisfies \( R_{\text{mac}}(W_1', W_2') = R_{\text{bc}}(T_{r1}^0, T_{r2}^0) \) and \( \tilde{R}_{r1}(W_1') \geq \tilde{R}_{r1}(T_{r1}^0) \).

Finding the optimal solution for the considered network optimization problem in this subcase is complicated due to the fact that the optimization over \( \{W_1, W_2\} \) and \( \{T_{r1}, T_{r2}\} \) cannot be decoupled. Therefore, the optimal solution needs to be found via an iterative optimization over \( \{W_1, W_2\} \) and \( \{T_{r1}, T_{r2}\} \). However, by using a well-designed algorithm, the optimal solution in the above subcase can be found efficiently. Details of finding the optimal solution in this subcase will be available in the corresponding journal paper [8].

It can be shown that the above three subcases cover all possible situations when \( R_{\text{mac}}^m(W_1^0, W_2^0) \geq R_{\text{bc}}(T_{r1}^0, T_{r2}^0) \).

B. The case when \( R_{\text{bc}}(T_{r1}^0, T_{r2}^0) > R_{\text{mac}}^m(W_1^0, W_2^0) \)

This case corresponds to the situation when the maximum achievable sum-rate of the MAC phase is less than the maximum achievable sum-rate of the BC phase. Similar to the previous case, there are also several subcases when \( R_{\text{bc}}(T_{r1}^0, T_{r2}^0) > R_{\text{mac}}^m(W_1^0, W_2^0) \). In order to classify this case into subcases, first introduce three variables \( \mu_1, \mu_2 \) and \( \mu_{\text{tmp}} \) such that

\[
\sum_{k=1}^{r_{\text{ri}}} \log \left( 1 + \frac{1}{\mu_1} \frac{\omega_{r1}(k)}{\sigma_1(k)^2} - 1 \right)^+ = \tilde{R}_{r1}(W_1^0) \quad (21)
\]

\[
\sum_{k=1}^{r_{\text{ri}}} \log \left( 1 + \frac{1}{\mu_2} \frac{\omega_{r2}(k)}{\sigma_2(k)^2} - 1 \right)^+ = \tilde{R}_{r2}(W_2^0) \quad (22)
\]

\[
\sum_{k=1}^{r_{\text{ri}}} \log \left( 1 + \frac{1}{\mu_{\text{tmp}}} \frac{\omega_{r1}(k)}{\sigma_{\text{tmp}}(k)^2} - 1 \right)^+ = R_{\text{mac}}^m(W_1^0, W_2^0) \quad (23)
\]

It can be shown that \( 1/\mu_{\text{tmp}} < \max\{1/\mu_1, 1/\mu_2\} \). Since \( R_{\text{bc}}(T_{r1}^0, T_{r2}^0) > R_{\text{mac}}^m(W_1^0, W_2^0) \), it is necessary that \( 1/\lambda_0 > 1/\mu_{\text{tmp}} \), where \( 1/\lambda_0 \) is obtained by performing the procedure in Step 1 of Table I. The following four subcases are possible.

Subcase I: \( 1/\mu_{\text{tmp}} \leq \min\{1/\mu_1, 1/\mu_2\} \). In this subcase, the optimal beamformer for source \( i \) is \( W_i^0 \). The relay finds its optimal strategy through solving the following convex problem

\[
\min_{\{T_{r1}, T_{r2}\}} \text{Tr}(T_{r1}T_{r1}^H + T_{r2}T_{r2}^H) \\
\text{s.t.} \quad \tilde{R}_{r1}(T_{r1}) + \tilde{R}_{r2}(T_{r2}) \geq R_{\text{mac}}^m(W_1^0, W_2^0) \quad (24a)
\]

The solution is in closed-form and can be found using (14) and (15) with

\[
\mu_{\text{tmp}} \text{ satisfies } (23).
\]

Subcase II: \( 1/\mu_1 \leq 1/\mu_{\text{tmp}} < 1/\mu_2 \leq 1/\lambda_0 \). In this subcase, the optimal beamformer for source \( i \) is also \( W_i^0 \). The relay finds its optimal strategy by solving the problem

\[
\min_{\{T_{r1}, T_{r2}\}} \text{Tr}(T_{r1}T_{r1}^H + T_{r2}T_{r2}^H) \\
\text{s.t.} \quad \tilde{R}_{r1}(T_{r1}) + \tilde{R}_{r2}(T_{r2}) \geq R_{\text{mac}}^m(W_1^0, W_2^0) \quad (26a)
\]

The solution is in closed-form. The optimal \( T_{rj} \) can be also found using (14) and (15) with

\[
\mu_j \text{ satisfies } (21) \text{ if } i = 1 \text{ and } \mu_{\text{tmp}} \text{ satisfies } (22) \text{ if } i = 2.
\]

The optimal \( T_{ri} \) can be shown to be (14) and (15) with

\[
p_{ri}(k) = \frac{1}{\mu_{\text{tmp}}} - \frac{\sigma_i(k)^2}{|\omega_{r1}(k)|^2}, k = 1, \ldots, r_{ri} \quad (28)
\]

where \( \mu_{\text{tmp}} \) satisfies

\[
\sum_{k=1}^{r_{ri}} \log \left( 1 + \frac{1}{\mu_{\text{tmp}}} \frac{1}{|\omega_{r1}(k)|^2} - 1 \right)^+ = \tilde{R}_{ri}(W_1^0) = R_{\text{mac}}^m(W_1^0, W_2^0) \quad (29)
\]
Table II: Algorithm for Deriving Optimal Solution for Subcase IV when $R^{bc}(T_{r_1}^0, T_{r_2}^0) > R^{mac}(W_1^0, W_2^0)$

1. Source $j$ decreases its transmission power from $P_j$ to $P_j' < P_j$. Solve (11a)-(11b) with $P_j$ substituted by $P_j'$. Obtain $R^{mac}(W_1, W_2)$, $R_{r_1}(W_1)$ and $R_{r_2}(W_2)$. If $R_{r_1}(W_1) > R_{r_2}(T_{r_2}^0)$ or $R_{r_2}(W_2) < R_{r_2}(T_{r_2}^0)$, $P_j = (P_j' + P_j)/2$ and repeat this step.

2. Search for $\lambda_j$ such that $\hat{R}_{r_1}(T_{r_1}) = \sum_{k=1}^{r_{r_1}} \log (1 + (\lambda_j \sigma_k^{(1)})^2 - 1)^+$ equals $\bar{R}_{r_1}(W_1)$. Set $p_{r_1}(k) = \frac{1}{\lambda_j} - \frac{\sigma_k^{(1)}}{\sigma_k^{(2)}}$. Allocate $P_r - \sum_{k} p_{r_1}(k)$ on $\mathcal{M}_{r_1}$ using waterfilling and obtain $\bar{R}_{r_1}(T_{r_1})$.

If $\bar{R}_{r_1}(T_{r_1}) > \bar{R}_{r_2}(W_2)$, search for $\lambda_i$ such that $\bar{R}_{r_i}(T_{r_i}) = \sum_{k=1}^{r_{r_i}} \log (1 + (\lambda_i \sigma_k^{(1)})^2 - 1)^+$ equals $\bar{R}_{r_i}(W_i)$. Set $p_{r_i}(k) = \frac{1}{\lambda_i} - \frac{\sigma_k^{(1)}}{\sigma_k^{(2)}}$ and go back to Step 1. If $\bar{R}_{r_i}(T_{r_i}) + \bar{R}_{r_i}(W_i) - R^{mac}(W_1, W_2) < -\epsilon$, set $P_j' = P_j'/2$ and go back to Step 1. If $\bar{R}_{r_i}(T_{r_i}) + \bar{R}_{r_i}(W_i) - R^{mac}(W_1, W_2) > \epsilon$, set $P_j' = (P_j' + P_j)/2$ and go back to Step 1.

Subcase III: $1/\mu_i \leq 1/\mu_m < 1/\lambda_0 < 1/\mu_j$ and there exists $\mu_j''$ such that

$$\sum_{k=1}^{r_{r_i}} \log \left( 1 + \frac{1}{\mu_j''} \left| \frac{\sigma_k^{(1)}}{\sigma_k^{(2)}} \right| \right) \geq R^{mac}(W_1^0, W_2^0) - \bar{R}_{r_i}(W_i^0)$$

and

$$\sum_{k=1}^{r_{r_i}} \left( \frac{1}{\mu_j''} - \frac{\sigma_k^{(1)}}{\sigma_k^{(2)}} \right) \geq \sum_{k=1}^{r_{r_i}} \left( \frac{1}{\mu_i - \frac{\sigma_k^{(1)}}{\sigma_k^{(2)}}} \right).$$

In this subcase, the optimal solutions for $p_{r_1}(k)$ and $p_{r_2}(k)$ are the same as those given by (27), (28), and (29).

Subcase IV: $1/\mu_i \leq 1/\mu_m < 1/\lambda_0 < 1/\mu_j$ and there is no $\mu_j''$ which satisfies conditions (30) and (31). In this subcase, the optimal sum-rate of the BC phase cannot achieve $R^{mac}(W_1^0, W_2^0)$ although $R^{mac}(W_1^0, W_2^0) < \bar{R}_{r_1}(T_{r_1}^0) + \bar{R}_{r_2}(W_2^0)$. This subcase becomes complicated since the optimization over the strategies of the sources and the relay cannot be decoupled. We propose an algorithm with low-complexity for efficiently finding a sub-optimal solution in this subcase, which is given in Table II. It can be shown that the solution found by this algorithm can be possibly optimal.

Summarizing the subcases, we have the following theorem.

**Theorem 3:** The optimal solution of the problem of sum-rate maximization with power consumption minimization in the network optimization scenario for the case that $R^{bc}(T_{r_1}^0, T_{r_2}^0) > R^{mac}(W_1^0, W_2^0)$ is equivalent to the optimal solution of the problems (24a)-(24b), and (26a)-(26c), in subcases I and subcase II or III, respectively. The sub-optimal solution in subcase IV can be found using the algorithm in Table II.

The complete procedure of solving the problem of sum-rate maximization and transmission power minimization for the scenario of network optimization is summarized in Table III.

Table III: Summary of the algorithm for network optimization.

1. The sources solve the MAC sum-rate maximization problem (11a)-(11b). Obtain $R_{r_1}^0(W_1^0)$ and $R_{r_2}^0(W_2^0)$.
2. The relay performs Step 1 of Table I. Obtain $p_{r_1}(k)$ and $M_{r_1}^0$, $i = 1, 2$. Calculate $R_{r_i}^0(T_{r_i}^0)$ according to (18).
3. Check if $R^{mac}(W_1^0, W_2^0) > R^{bc}(T_{r_1}^0, T_{r_2}^0)$. If yes, proceed to Step 4. Otherwise, proceed to Step 5.
4. Sources find their optimal strategy from solving the power minimization problem (19a)-(19c) for subcases I and II while the relay’s optimal strategy is to use $T_{r_1}^0$ and $T_{r_2}^0$.
5. Determine the subcase based on $\mu_1$, $\mu_2$, $\mu_m$, and $\lambda_0$. For subcase I and subcases II or III, the optimal beamformer for source $i$ is $W_i^0$ and the relay minimizes its transmission power via solving the problems (24a)-(24b), and (26a)-(26c), respectively. For subcase IV, perform the algorithm in Table II for deriving the sub-optimal strategies for the sources and the relay.

V. SIMULATIONS

In this section, we show the performance of the proposed algorithms for relay optimization in Table I and for network optimization in Table II. The general setup is as follows. The elements in channels $H_{r_i}$ and $H_{r_i, vi}$ are generated from complex Gaussian distributions with zero mean and unit variance. Noise powers $\sigma_i^2, \forall i$ are set to 1. The rates $R^{mac}(W_1, W_2)$, $R^{bc}(T_{r_1}, T_{r_2})$, $R_{r_1}(W_1)$, and $R_{r_1}(T_{r_1})$ are briefly denoted as $R^{mac}$, $R^{bc}$, $R_{r_1}$, and $R_{r_1}(T_{r_1})$, respectively.

Fig. 1 compares the BC phase rates for the optimal solution of the problem (12a)-(12b) (which does not minimize the power consumption) and the optimal solution of the considered relay optimization problem which minimizes the relay power consumption versus the relay power limit $P_r$ for one channel realization. The specific setup for this simulation is as follows. The number of antennas $n_1, n_2$, and $n_r$ are set to be 6, 5 and 8, respectively. Power limits for the sources are $P_1 = P_2 = 3$. The MAC phase rates for this channel realization are 19.82 for $R^{mac}$, 11.84 for $R_{r_1}$, and 10.19 for $R_{r_2}$. In this figure, $R_{r_1}$ is the rate from the relay to the source $i$ for the optimal solution of the convex problem (12a)-(12b) and $R_{r_1}$ is the rate from the relay to the source $i$ for the optimal solution of the considered relay optimization problem obtained using the algorithm in Table I. It can be seen from the figure that the algorithm in Table I generates the same rates as those obtained from solving the problem (12a)-(12b) when $P_r$ is small. The reason is that $R_{r_1}$ is small when $P_r$ is below certain threshold and as a result (13a) and (13b) are always satisfied. As $P_r$ increases, the objective function (12a) is bounded by $R^{mac}$ while the relay’s transmission power is necessarily minimized in the solution of the problem (12a)-(12b). In Fig. 1, the optimal solution of the problem (12a)-(12b) always uses all available power while the optimal solution found using the proposed algorithm does not use more power after $P_r$ exceeds 4.16, at which point $R^{bc}$ achieves $R^{mac}$.

Fig. 2 shows instantaneous $R^{lw}$, $R^{mac}$ and $R^{bc}$ versus the number of iterations when the algorithm in Table II is performed for network optimization subcase IV when $R^{bc}(T_{r_1}^0, T_{r_2}^0) > R^{mac}(W_1^0, W_2^0)$. The specific setup for this simulation is as follows. Number of antennas at source 1,
source 2, and the relay are 6, 4, and 8, respectively. Power limits are set as $P_1 = 2$, $P_2 = 8$, $P_r = 8$. The channel realization leads to the result that $1/\lambda_0 = 1.12$, $1/\mu_1 = 1.83$, $1/\mu_2 = 0.61$ and $1/\mu_{m1} = 1.04$. It can be seen from the figure that the algorithm searches for the maximum $R_{mac}$ such that it can be achieved by $R^{bc}$. During this search, $R^{tw}$ gradually achieves its maximum value. Fig. 3 shows the instantaneous $\hat{R}_1$, $\bar{R}_1$, and $\hat{R}_{r1}, \forall i$ and the power usage of the relay denoted as $P_r$ and source 2 denoted as $P_2$ (source 1 always uses all power). Two observations can be drawn from Fig. 3. First, $\hat{R}_{r2} = \hat{R}_{1r}$, and $\hat{R}_{r1} < \hat{R}_{2r}$, for the solution because the sum-rate is bounded by $R^{bc} = R_{mac} < \hat{R}_{1r} + \hat{R}_{2r}$. Second, source 2 does not use all available power in the solution since $P_2 < P_2$.

VI. CONCLUSION

In this work, we have efficiently solved the problem of sum-rate maximization with minimum transmission power consumption for DF MIMO two-way relaying. As a result, the considered two-way relay system achieves both high spectral efficiency and high power efficiency at the same time through optimizing the power allocation/beamforming strategies of the participating nodes. For the considered scenario of relay optimization, the proposed algorithm allows the relay to decide its optimal power allocation/beamforming in at most 6 steps. For the scenario of network optimization, for all except two subcases, we have shown that the problem can be simplified to different convex problems, which have closed-form solutions in some subcases. For a remaining subcase, we proposed a low-complexity algorithm that efficiently finds the sub-optimal solution in a few iterations.

REFERENCES